UNIT 5 VOCABULARY: DECIMAL NUMBERS

1.1. Decimal numbers

Decimal numbers are used in situations in which we look for more precision than whole numbers provide.

In order to do that, we define:

- **A tenth:** each part that we get when we divide something in **ten equal parts**. It is represented by 0.1 = $\dfrac{1}{10}$

- **A hundredth:** each part that we get when we divide something in **a hundred equal parts**. It is represented by 0.01 = $\dfrac{1}{100}$

- **A thousandth:** each part that we get when we divide something in **a thousand equal parts**. It is represented by 0.001 = $\dfrac{1}{1,000}$

One

Tenths

Hundredths

Thousandths

1.2. Place value

In the decimal number system ,the value of a digit depends on where it is placed:

> ⚠️ **Adding extra zeros to the right of the last decimal digit does not change the value of the decimal number.**
> **For example, 3.24 = 3.240 = 3.2400 ...**

1.3. How to read decimal numbers

Decimal numbers are read with each figure separate. We use a full stop (called **"point"**), not a comma, before the decimal places.

You can also read the **full number after the decimal point** and then say **the word for the last place value**.

Examples:

2.34 ➜ two **point** three four **or** 2 point thirty-four hundredths.

3.375 ➜ three **point** three seven five **or** three point three hundred and seventy-five thousandths.

0.75 ➜ (nought or zero) point seven five **or** seventy-five hundredths.

Exercises.

1. Read the following numbers:

120,000.321	453.897	700,560	5,542.678987
34.76	0.54	0.054	8,275.4

2. Write with words the following numbers:

   21.456

   0.77

   0.0089

   5.7254

1.4. Converting fractions into decimals

In unit 4 we studied that every fraction can be expressed as a decimal number. To convert a fraction into a decimal, you just have to divide numerator by denominator.
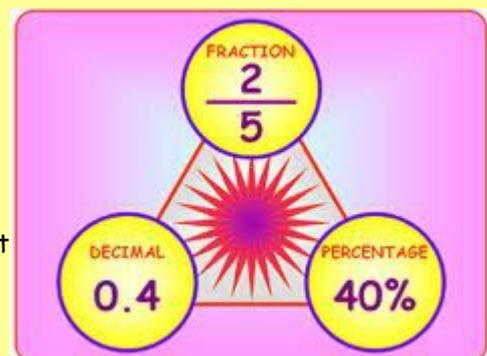
The quotient of a fraction can be:

- **Integers:** no decimal part. For example, $\dfrac{6}{2}=3$

- **Exact (or terminating) decimals:** decimal numbers that end (or terminate).

  For example, $\dfrac{3}{10}=0.3$ or $\dfrac{5}{4}=1.25$

- **Repeating or recurring decimal numbers**: decimal numbers that have a recurring pattern of a single or multiple digits.

  For example, $\dfrac{1}{3}=0.333333...=0.\hat{3}$ $\qquad \dfrac{2}{11}=0.\widehat{18}...$ $\qquad \dfrac{31}{12}=2.58\hat{3}$

$$\frac{1}{3} \ = \ 0.333... \ = \ 0.\dot{3} \ = \ 0.\overline{3}$$

Fraction   Ways to show recurring decimals

Exercise. Convert to a decimal:

a) $\dfrac{1}{4}$ b) $\dfrac{3}{20}$ c) $\dfrac{13}{5}$ d) $\dfrac{17}{6}$ e) $\dfrac{131}{11}$

1.5. Converting decimals into fractions

But decimals can be expressed as fractions. To convert **an exact decimal** into a fraction, you have to follow these steps:
- The **numerator** is formed by the **digits without the decimal point**.
- The **denominator** is the number formed by **"1" and as many zeros as decimal figures the number has**.
- **Reduce**, if possible.

   For example, $2.75 = \dfrac{275}{100} = \dfrac{11}{4}$ $0.6 = \dfrac{6}{10} = \dfrac{3}{5}$ $3.002 = \dfrac{3002}{1,000} = \dfrac{1501}{500}$

Exercise. Write these decimal numbers as fractions:

a) 0.4   b) 1.2   c) 0.045   d) 2.625

1.6. Representation of decimals on the Number Line

To represent a decimal on a number line, divide each segment of the number line into ten equal parts.
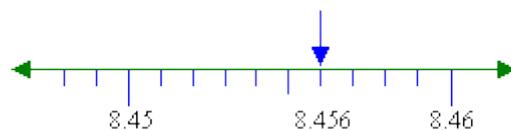
For example, to represent 8.4 on a number r line, divide the segment between 8 and 9 into ten equal parts.
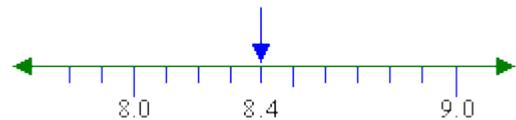
Likewise, to represent 8.45 on a number line, divide the segment between 8.4 and 8.5 into ten equal parts.
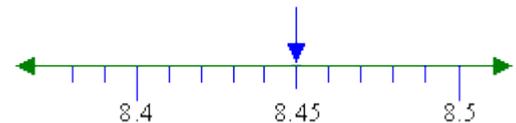
Similarly, we can represent 8.456 on a number line by dividing the segment between 8.45 and 8.46 into ten equal parts.

Exercise. Write the decimal number that the arrow points at in the following diagrams:
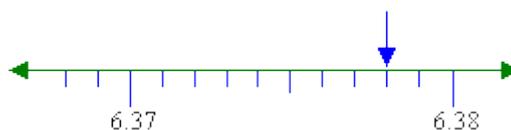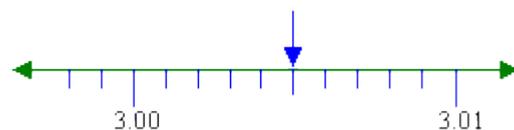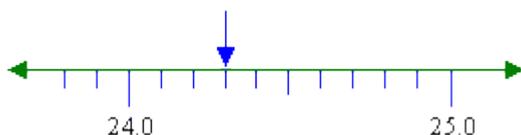
2.1. Addition and subtraction of decimals

To add or subtract decimals, **line up the decimal points** and then follow the rules for adding or subtracting whole numbers, **placing the decimal point in the same column**.

When one number has more decimal places than another, use 0's to give them the same number of decimal places.

For example, to add 528 and 7.49
- Line up the decimal points and add 0s on the right of the first number.
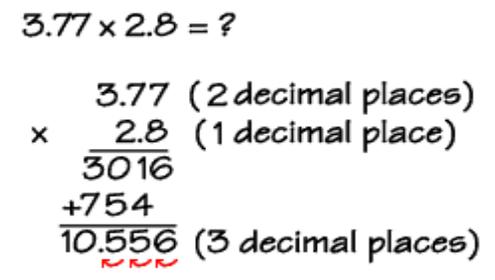- Then add.

$$528 + 7.49$$

Line up the decimal points...

$$
\begin{array}{r}
528.00 \\
+\ \ \ 7.49 \\
\hline
535.49
\end{array}
$$
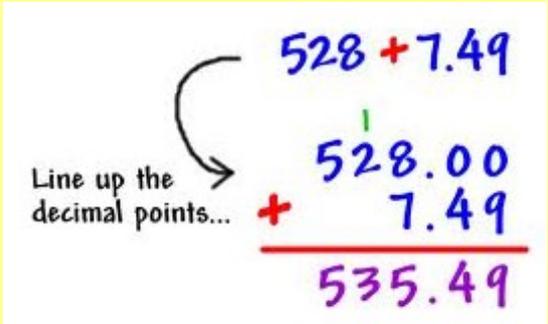
2.2. Multiplication of decimal numbers

Multiplying decimals is just like multiplying whole numbers. The only extra step is to decide how many digits to leave to the right of the decimal point. To do that, **add the numbers of digits to the right of the decimal point in both factors**.

For example, to multiply 3.77 by 2.8:

$$3.77 \times 2.8 = ?$$

$$
\begin{array}{r}
3.77 \ \text{(2 decimal places)} \\
\times \quad 2.8 \ \text{(1 decimal place)} \\
\hline
3016 \\
+754 \quad \\
\hline
10.556 \ \text{(3 decimal places)}
\end{array}
$$

Exercises. Work out:

   a) $5.6 \cdot 6.9$ **b)** $12.37 \cdot 76.78$ **c)** $-4.66 \cdot 4.7$ **d)** $0.345 \cdot (32.4 - 4.67)$

3.1. Dividing whole numbers, with decimals

To get decimals in a division, continue the whole division adding zeros to the right of the number being divided until you get the amount of decimal digits required.

For example, divide 235:6 until the hundredth:

$$
\begin{array}{r|l}
235 & 6 \\
55 & 39.16 \\
10 & \\
40 & \\
4 &
\end{array}
$$
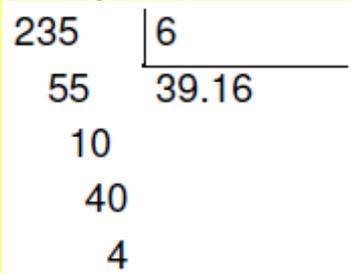
Exercise. Calculate with two decimal digits:

   a) $56 \div 7$ **b)** $7634 \div 34$ **c)** $-679 : 32$ **d)** $9783 : 127$

3.2. Dividing decimals by decimals

To **divide by a decimal by another decimal**, multiply the divisor by a power of 10 great enough to obtain a whole number. **Multiply the dividend by that same power of 10**. Then the problem becomes one involving division by a whole number instead of division by a decimal.

For example:

```
1 9 1 4 |1.5      →  1 9 1 4 0 |15
              0 4 1        1 2 7 6
              1 1 4
              0 9 0
                0 0
```

Exercise. Calculate with three decimal digits:

   a) 56.7 ÷ 2.34   **b)** 1432.3 ÷ 0.42   **c)** – 12.34 ÷ 3.5   **d)** 1 ÷ 1.2

3.3. Order of operations

Once again!!
When you have several operations to do, which one do you calculate first?
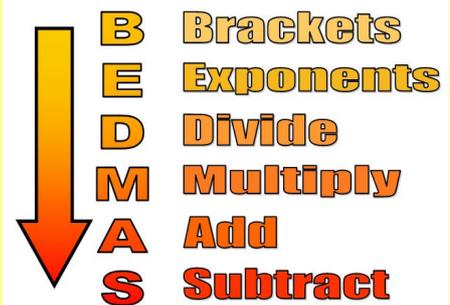
We work out operations in this order:

BRACKETS

EXPONENTS (Powers, roots, etc)

DIVISION and **M**ULTIPLICATION (working from left to right)

ADDITION and **S**UBTRACTION (working from left to right)

That makes **BEDMAS!**

B Brackets
E Exponents
D Divide
M Multiply
A Add
S Subtract

4.1. Approximations

An approximation of a number is a representation of that number that is not exact, but still close enough to be useful.
Rounding off a decimal number to a given number of decimal places is the quickest way to approximate a number.

For example, if you want to round off 2,6525272 to three decimal places, you would:

Step 1: Mark off the required number of decimal places. **2,652|5272**
Step 2: Check the next digit to see if you must round up or round down. Remember: if the next digit is 5 or more, you must round up, and if it is 4 or less, you must round down. **2,652|5272** must be rounded up.
Step 3: Write the final answer. **2,653** rounded to 3 decimal places.

> **REMEMBER!**
> When the digit **5, 6, 7, 8, or 9** appears in the ones place, **round up**;
> when the digit **0, 1, 2, 3, or 4** appears in the ones place, **round down**.

Exercises.

1. Round off these distances to the nearest 100 m (to one decimal place):
   a) 5.768 km   c) 8.48 km   e) 17.685 km
   b) 9.039 km   d) 8.41 km   f) 17.658 km

2. Round off:
   a) 1.17 to the nearest tenth   d) 12.87 to the nearest unit
   b) 2.375 to the nearest hundredth   e) 151.504 to the nearest hundred
   c) 0.7084 to the nearest thousandth   f) 7478 to the nearest thousand

3. The height of each person has been rounded to the nearest 10 cm.

Beth 170 cm   Fowsia 150 cm   Craig 140 cm   Jie 160 cm   Ian 180 cm   Karim 190 cm

The actual heights of the six people are:

1.44 m   1.54 m   1.61 m   1.65 m   1.84 m   1.85 m

Match each person with their actual height.

4.2. Word problems

1) Ellen earns 137.40 per week. After 4 weeks, she gets an extra payment of 24.75. She spends 354.60 in this period. How much money does she save?

2) A student studies a total time of 4 h 35 min and during this time he writes 100 min. How long, in hours, does he study without writing?

3) Susan is cooking a cake and uses 1.35kg of flour, 0.37kg of sugar, 3 eggs that weigh 80g each and 240g of milk. What is the weight of the mixture ?
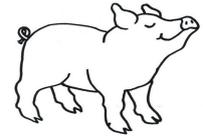
4) I buy 7 kilograms of meat and pay € 53.55 . How much does the kilogram of meat cost?

5) Henry has € 83.40 . He buys four tickets for the cinema at € 6.50 each and 2 bags of popcorn at € 2.25 each. How much money has he got left?

6) A breeder gives to each pig 0.65 kg of food for every 4 kg of body weight. There are 4 pigs of 75.8 kg, 56.4 kg, 75.4 kg and 89.3 kg. How much food must be prepared in total?

7) In a restaurant, 7 friends are having a meal. The bill is £173.6 and each person contributes with £25.50. What tip does the waiter receive?

8) A company produces items that are sold for £13.63 each. The daily production is 1275 items and the cost of production is £11,324.50. What is the daily income for the company?
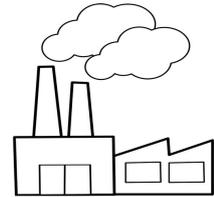
9) Fernando Alonso can travel at 167.33 mph in his McLaren Formula 1 car. How far can he go in 4 hours?

10) Ted was 88.53 cm tall when he was 3 years old. By the time he was 19, Ted had grown a further 85.92 cm. How tall was he when he was 19?